UNIT 9: PROBABILITY

Day 1: Counting Principle and Permutations

3-DIGIT CODES

Area codes were invented in the 1940s when the Bell Telephone Company realized they could not hire enough operators to handle all of the long-distance calls that were being made.

Bell researchers developed a system of 3-digit codes that would automatically route calls to long distance.

We now call this an area code.

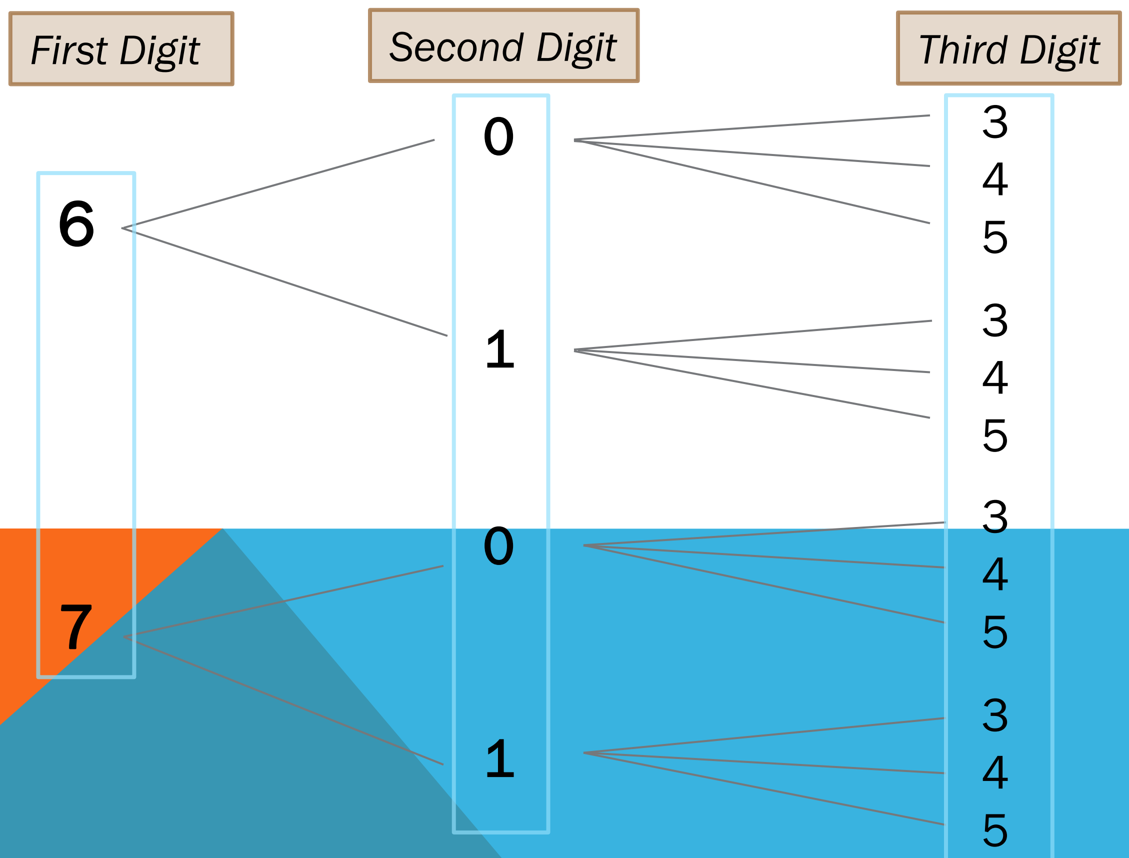
EXAMPLE 1

Suppose Texas is adding a new area code.

The first digit must be a 6 or 7, the second digit must be a 0 or 1, and the third digit can be a 3, 4 or 5.

How many area codes are possible?

(show a tree diagram mapping the number for all the combinations)



There are 12 choices for a new area code.

LUNCH DILEMMA

Suppose you are at your local Sonic Drive-In for lunch. You want to choose 1 sandwich, 1 side, 1 drink and 1 dessert. Below are the different options. How many meal possibilities can you make?

Entrees	Sides	Drinks	Dessert
Hamburger	Fries	Soda	Ice Cream
Chicken Sandwich	Tater Tots	Tea	Milk Shake
Chili Cheese Dog	Onion Rings	Slush	Brownie
Chicken Wrap		Water	Cookie

THE TREE DIAGRAM HELPS SHOW THE NUMBER OF POSSIBLE OUTCOMES

Entrees	Sides	Drinks	Dessert
Hamburger	Fries	Soda	Ice Cream
Chicken Sandwich	Tator Tots	Tea	Milk Shake
Chili Cheese Dog	Onion Rings	Slush	Brownie
Chicken Wrap		Water	Cookie

Each Entrée has 3 possible side choices, so $4(3) = 12$ entrée/side combinations

Each Entrée/Side has 4 possible drink choices, so $12(4)=48$ entrée/side/drink combos

Each Entrée/Side/Drink combo has 4 Dessert choices, so $48(4)= 192$ total choices

THE COUNTING PRINCIPLE:

When there are *m* ways to do one thing,
and *n* ways to do another,
then there are *m*n* ways of doing both

So our Sonic Example would be:

4 * 3 * 4 * 4 = 192

GROUP ACTIVITY

Each group pick a scenario and determine the possible outcomes using a tree diagram and the Counting Principle. Show your work and be ready to present to the class!!

Group 1	Group 2	Group 3	Group 4
Murder Mystery	Jeans Store	Ice Cream Shoppe	Party City
Group 5	**Group 6**	**Group 7**	**Group 8**
Car Dealership	Movie Theater	Pizza Parlor	Freebirds!

MURDER MYSTERY

Suspects	Rooms	Weapons
Colonel Mustard	Kitchen	Rope
Professor Plum	Study	Lead pipe
Beth	Library	Knife
Miss Scarlet	Hall	Wrench
Mrs. White	Garden	Candlestick
Mr. Green	Dining room	shovel
	Ballroom	
	Conservatory	
	Billiard room	

6*9*6 = <u>324</u> total outcomes

JEANS STORE

Sizes	Fits	Lengths
3	Boot cut	Short
5	Skinny	Regular
7	Super Skinny	Long
9	Jeggings	
11		
13		
15		

7*4*3 = **<u>84</u>** outcomes

ICE CREAM SHOPPE

Flavor	# of Scoops	Container
Vanilla	1	Cup
Chocolate	2	Waffle Cone
Strawberry	3	Chocolate Dipped Cone
Mint Chocolate Chip		
Chocolate Chip Cookie Dough		
Cookies ‘n’ Crème		
Rocky Road		

63 outcomes

PARTY CITY

Theme	Decorations	Cakes
Birthday	Balloons	Classic
Luau	Streamers	Ice Cream Cake
Super Bowl	Napkins	Cookie Cake
4th of July	Plates	Cupcakes
Costume	Confetti	
Toga	Lights	
	Banner	

168 outcomes

CAR DEALERSHIP

Type	Make	Color	Interior
Car	Ford	Black	Leather
Truck	Chevy	White	Cloth
SUV	Honda	Red	
	Toyota	Silver	
	Infiniti	Yellow	
	BMW		
	Mercedes		

<u>210</u> outcomes

MOVIE THEATER

Popcorn	Drink	Candy
Small	Small	Sour Patch Kids
Medium	Medium	Reese’s
Large	Large	Twizzler’s
Extra Large Bucket		Junior Mints
		Goobers
		Mike & Ike’s

72 outcomes

PIZZA PARLOR

Size	Crust	Toppings
Small	Hand-tossed	Pepperoni
Medium	Pan	Sausage
Large	Chicago Style	Hamburger
	New York Style	Onion
		Bell Pepper
		Black Olives
		Mushrooms

<u>84</u> outcomes

FREEBIRDS!

Type	Meat	Toppings
Burrito	Chicken	Cheese
Taco	Steak	Sour Cream
Bowl	None	Corn
Salad		Beans
		Salsa
		Onions
		Guacamole

<u>84</u> outcomes

Now that you can find the total number of outcomes, let's move on to *Permutations*.

PERMUTATIONS AND COMBINATIONS

Mary decides to flip a coin and record the outcomes. Every 3 flips she records what happened. In the table below this is what she has recorded so far.

	Flip 1	Flip 2	Flip 3
Outcome 1	Heads	Tails	Heads
Outcome 2	Tails	Heads	Heads
Outcome 3	Heads	Heads	Tails

How are the three outcomes similar?

How are the three outcomes different?

	Flip 1	Flip 2	Flip 3
Outcome 1	Heads	Tails	Heads
Outcome 2	Tails	Heads	Heads
Outcome 3	Heads	Heads	Tails

1. Which outcome describes a coin flip that resulted in heads the first flip, heads the second flip and tails the third flip?

Outcome 3

Did the order matter when determining the answer to question 1? Why or why not?

Yes. Specific order asked for in question.

2. Which outcome might describe a coin flip that resulted in heads twice and tails once?

All 3 outcomes

Did the order matter when determining the answer to question 2? Why or why not?

No. Order was not specified in the question.

Can the three outcomes be considered the same if the order of the elements matters? Justify your answer.

Can the three outcomes be considered the same if the order of the elements *doesn't* matter? Justify your answer.

FACTORIALS!

You have 6 Beanie Babies on your bed.

How many ways can you put them in order?

That's a lot of multiplying!!

$\underline{1^{st}}$ $\underline{2^{nd}}$ $\underline{3^{rd}}$ $\underline{4^{th}}$ $\underline{5^{th}}$ $\underline{6^{th}}$

There is a faster way…

Any of the 6 could be first.

2nd position has one of the 5 remaining bears.

Third position has 4 to choose from.

And so on.

Use the counting principle:

$\underline{6} * \underline{5} * \underline{4} * \underline{3} * \underline{2} * \underline{1} = 720$ ways

FACTORIALS!

French Mathematicians Abogast and Kramp both generated the idea of the *factorial*.

It is a simpler way to show the product of the numbers from 1 to the number you want.

Remember the 6 Beanie Babies?

6*5*4*3*2*1 can be represented by 6!

6! means multiply 6 by all of the numbers before 6.

It is represented with an

BUT WAIT!! IT GETS BETTER!!!

There's a button on your calculator for **!**

Try 4! the long way. 4*3*2*1 Press enter.

Now, type in 4, Math, arrow to the PRB tab. (That's probability, btw.)

The 4^{th} choice is factorial. Press 4.

Your screen should now show 4!

Press enter.

Do your two answers match?

Permutations

An ordered arrangement of items is called a permutation.

Clue words: arrangement, order, lists, schedule

	Flip 1	Flip 2	Flip 3
Outcome 1	Heads	Tails	Heads
Outcome 2	Tails	Heads	Heads
Outcome 3	Heads	Heads	Tails

Consider the "Flipping a Coin" example.

When we specified what the first, second and third flip should be the order of the elements mattered, therefore it was a permutation.

Environmental Club
Officers Ballot

President________________

Vice President____________

Treasurer________________

The environmental club is electing a president, a vice president, and a treasurer.

How many different ways can the officers be chosen from the 10 members who are running for office?

Pres VP Treas

Any of the 10 can be president. So the first spot has 10 choices.

10 VP Treas

Now, there are only 9 choices for the vice president spot.

10 9 Treas

That leaves 8 people for the treasurer position.

10 9 8

Will this answer be 8! ?

From the counting principle, 10*9*8 = 720 different ways.

What if, out of the 10 people running for office from the environmental club, there were 8 positions available?

How many permutations would there be?

10 * 9 * 8 * 7 * 6 * 5 * 4 * 3

This isn't 10!, but can it be represented using factorials?

$$\frac{\#Total!}{(\#Total - \#Want)!} \rightarrow \frac{10!}{(10-8)!}$$

Use your calculator to solve

1,814,400 different ways to fill the 8 positions

Hmmm. Will this work with the original problem?
10 members, 3 positions?

$$\frac{\#Total!}{(\#Total - \#Want)!} \rightarrow \frac{10!}{(10-3)!}$$

720 possible ways 3 of 10 can be arranged.

Isn't this better than multiplying it all out one number at a time?

No?

Calculating Permutations: There is a button for that, too!

Permutations - a different arrangement of the same 3 members is a different result, so the order matters.

n P r is the calculator function for permutations.

Total number of things (n)

Chosen number of things (r)

$$\frac{\#Total!}{(\#Total - \#Want)!} \quad \textit{means} \quad \frac{n!}{(n-r)!}$$

"n" represents the number of total elements and "r" represents the number you are choosing. We have 10 total members running for office and we are choosing 3 for a specific office.

It is written like this: 10 P 3

To calculate the number of permutations, in your calculator press "10", MATH, cursor over to PRB and select 2, then press "3"

This should be on your screen: 10 P 3

Now, press enter to solve.

Try This:

An ice cream shop sells sundaes as 2 layers, 4 layers and 5 layers of flavors. The shop carries 65 flavors in all.

a) How many different **4** layer sundaes can you make if you *do not repeat* any flavors?

$_{65}P_{4}$

16,248,960
different sundaes

b) Why is this a permutation?

Only one flavor can be first, leaving 64 to be second, etc. And no repeating flavors!!

c) Which type of sundae would give you the greatest number of choices if you do not repeat flavors?

$_{65}P_{2} = 4160$

$_{65}P_{4} = 16,248,960$

$_{65}P_{5} = 991,186,560$

5 layers has the greatest number of choices.

DAY 2: COMBINATIONS

Combinations

An <u>unordered</u> collection of items is called a combination.

Clue words: Group, committee, sample

Remember the "Flipping a Coin" example?

	Flip 1	Flip 2	Flip 3
Outcome 1	Heads	Tails	Heads
Outcome 2	Tails	Heads	Heads
Outcome 3	Heads	Heads	Tails

When we **did not specify the order** of the elements, the outcomes always contained the same elements: two Heads and one Tail.

You have a new lock on your phone that uses a 2 letter code out of A, B, C, D,E with no repeats.

How many combinations are there if the order you choose doesn't matter?

AB AC AD AE
~~BA~~ BC BD BE
~~CA~~ ~~CB~~ CD CE
~~DA~~ ~~DB~~ ~~DC~~ DE
~~EA~~ ~~EB~~ ~~EC~~ ~~ED~~

10 ~~20~~ combos

5*4*3*2*1 are the total number of choices.

You only want two of them, so 2*1

Divide these to take out the 3 "extra" choices.

$$\frac{5*4*\cancel{3}*\cancel{2}*\cancel{1}}{\cancel{3}*\cancel{2}*\cancel{1}}$$

That leaves 5*4.

If you stop here, you have a *permutation*.

Now, remove the choice order, 2*1

$$\frac{5*4}{2*1}$$

Let's remove the doubles since order doesn't matter

There are **10** combinations.

Your English class requires you to choose 4 books to read over Summer Break from a list of 12.

How many different ways are there in which you can choose the books?

$$\frac{12!}{(12-4)!\ 4!}$$

Take out the 8 extra choices

Take out the order choice for each spot

Calc strokes: **12! / ((12-4)! 4!)**

495 ways to choose

Calculating Combinations: There's a button for that, as well!

The book examples involves combinations because the books chosen are what is important, not the order in which you read them.

n C r is the calculator function for combinations.

Total number of things (n)

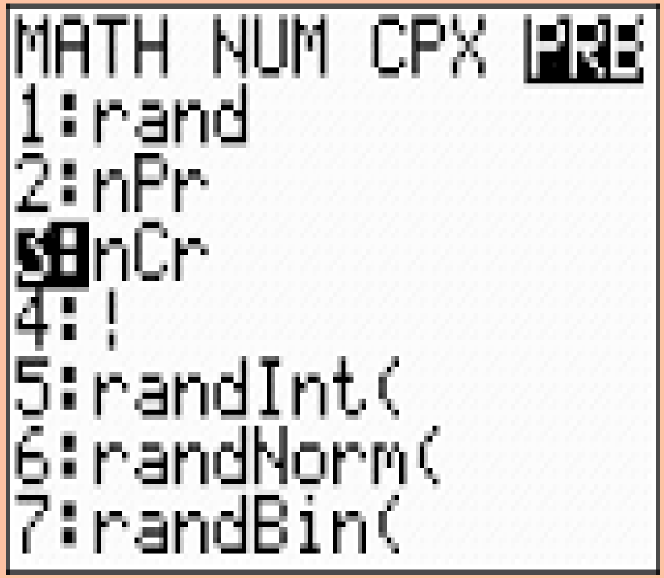
Chosen number of things (r)

$$\frac{\#Total!}{\#Want!\ (\#Total - \#Want)!} \quad \text{means} \quad \frac{n!}{r!\ (n-r)!}$$

"n" represents the number of total elements and "r" represents the number you are choosing. We have 12 total books to select from and we are choosing 4 to read.

It is written like this: **12 C 4**

To calculate the number of combinations, in your calculator press "12", MATH, cursor over to PRB and select 3, then press "4", ENTER



This should be on your screen: 12 C 4

Now, press enter to solve.

A service club has 20 freshmen. 7 of the freshmen are to be chosen to be on a clean-up crew for the town's annual picnic.

a) How many different ways are there to make the crew?

20 C 7

77,520

b)Why is this a combination and not a permutation?

Just choosing 7 not ordering 7 positions.

c) The club also has 18 Sophomores. If 7 Sophomores are chosen to join the 7 Freshmen in the cleaning crew, how many ways can the crew be made now?

There is a freshman choosing and a sophomore choosing. Do them separately and multiply the outcomes.

20 C 7 * 18 C 7

77,520 * 31,824

2,466,996,480

crew combos

Permutation or Combination?

Then, solve.

- The number of ways you can choose a group of 3 puppies from the animal shelter when there are 20 breeds to choose from (assume you don't choose the same breed twice).

C 1140

- The number of seven-digit phone numbers that can be made using the digits 0-9.

P 604,800

- The number of ways you could award 1st, 2nd, and 3rd place medals for the science fair where 52 students competed.

P 132,600

- The number of ways a committee of 3 could be chosen from a group of 20.

C 1140

- The number of ways a president, vice-president, and treasurer could be chosen from a group of 20.

P 6840

- A standard deck of cards has 52 playing cards. How many different 5-card hands are possible?

C 2,598,960

- There are 13 people on a softball team. How many ways are there to choose 10 players to take the field?

C 286

- There are 5 people on a bowling team. How many ways can you choose your bowling team captain and team manager?

P 20

- A pizza parlor has a special on a three-topping pizza. How many different special pizzas can be ordered if the parlor has 8 toppings to choose from? (no repeats)

C 56

- A pizza shop offers 12 wing flavors. How many different 3- flavor wing plates can be formed if order matters?

P 1320

- Find the number of possible committees of 3 people that could be chosen from a class of 30 students?

C 4060

- There are 11 seniors on a football team that are being considered as team captains. If there will be 4 team captains, how many different ways can the seniors be chosen as captains?

C 330

- Your English teacher has asked you to select 3 novels from a list of 10 to read as an independent project. In how many ways can you choose which books to read?

C 120

- You download 11 songs on your IPOD. If you select random shuffle, how many different orders could the 11 songs be played?

P 39,916,800

- 14 athletes are competing in the X-games. In how many possible ways can the athletes get gold, silver, bronze, and honorable mention?

P 24,024

- There are 30 students in the classroom. Six of them are to be chosen to clean up the room. How many different ways are there to choose the 6 students to clean up?

C 593,775

- There are 11 people on a baseball team. How many different ways can a pitcher and a catcher be chosen?

P 110

- How many different numbers can be made using any three digits of 12,378?

P 60

- How many different ways can you arrange 10 CDs on a shelf?

P 3,628,800

- A professional basketball team has 12 members, but only five can play at one time. How many different groups of players can be on the court at one time?

C 792

- Megan has 4 different skirts and 8 different blouses to choose from. How many outfits are possible if she chooses 1 skirt and 1 blouse?

C 32

DAY 3: ALL TYPES PROBABILITY

Previously, in *middle* school…

You draw on marble out of jar containing 4 red ones, 2 blue ones, and 6 white ones.

What is the probability that you draw a red marble? **4 red out of 12 total**

Remember…

$$P\text{ (event occurring)} = \frac{\#\text{desired outcomes}}{\#\text{ total outcomes}}$$

$$P\text{ (red)} = \frac{4}{12} = \frac{1}{3}$$

You have a 1 in 3 chance of grabbing a red one.

What percentage is that? 33.33%

INTRO TO PROBABILITY

When given a die, what is the probability of rolling a number greater than 3?

Roll #	Outcome
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

- Get one die and a partner.
- Make a chart.
- Record the outcome of 10 rolls of the die.

Using your data from the table, what was the probability that you rolled a number greater than 3?

Why is your probability different from your neighbors'?

How can you determine the probability of rolling a number greater than 3, *without* having to do the experiment?

P (event occurring) = #desired outcomes / # total outcomes

$$P(\text{event occurring}) = \frac{\#\text{desired outcomes}}{\#\text{total outcomes}}$$

There are three numbers greater than 3 out of the six possible. 4, 5, and 6

$$P(\#\text{greater than } 3) = \frac{3}{6} = \frac{1}{2}$$

Was that what happened on your 10 rolls?

Were half of your outcomes greater than 3?

So, what is the difference between:

theoretical probability and ***experimental probability***?

Try this: Determine the probability of each scenario.

1. What is the probability of choosing a king from a standard deck of playing cards?

$$P(\text{King}) = \frac{4}{52} = \frac{1}{13}$$

2. What is the probability of choosing a green marble from a jar containing 5 red, 6 green and 4 blue?

$$P(\text{green}) = \frac{6}{15} = \frac{2}{5}$$

3. What is the probability of choosing a marble that is not blue in problem 2?

$$P(\text{not blue}) = \frac{9}{15} = \frac{3}{5}$$

4. What is the probability of getting an odd number when rolling a single 6-sided die?

$$P(\text{odd}) = \frac{3}{6} = \frac{1}{3}$$

INDEPENDENT AND *CONDITIONAL* PROBABILITY

You have three peppermint and two butterscotch candies in front of you.

You close your eyes and pick one.

What is the probability that you will choose a peppermint?

P(*peppermint*) = $\frac{3}{5}$

You put it back.

What is the probability that you will choose a butterscotch?

P(*butterscotch*) = $\frac{2}{5}$

Why aren't these probabilities equal to each other?

What is the probability that you pick a butterscotch and then a peppermint?

What you do with the 3/5 and 2/5??

THE "*AND*" RULE :

$$P(A \text{ and } B) = P(A) * P(B)$$

What is the probability that you pick a butterscotch and then a peppermint?

$$P(b \text{ and } p) = \frac{2}{5} * \frac{3}{5}$$

$$\frac{6}{25}$$

What is the probability that you pick a peppermint and then another peppermint?

$$P(p \text{ and } p) = \frac{3}{5} * \frac{3}{5}$$

$$\frac{9}{25}$$

Solve using the "and" rule.

A jar contains 6 red balls, 3 green balls, 5 white balls, and 7 yellow balls. Two balls are chosen from a jar, with replacement. What is the probability that both balls chosen are green?

P(g and g) = 3/25 * 3/25

= 9/625

Two cards are chosen at random from a deck of 52 cards with replacement. What is the probability of choosing two kings?

P(k and k) = 1/13 * 1/13

= 1/169

You have three peppermint and two butterscotch candies in front of you... again.

You close your eyes and pick one and eat it. You do that again.

What is the probability that you chose a butterscotch both times?

Is this an "and" type of problem?

If you eat one, how many are left for the second draw?

How does that change the second fraction?

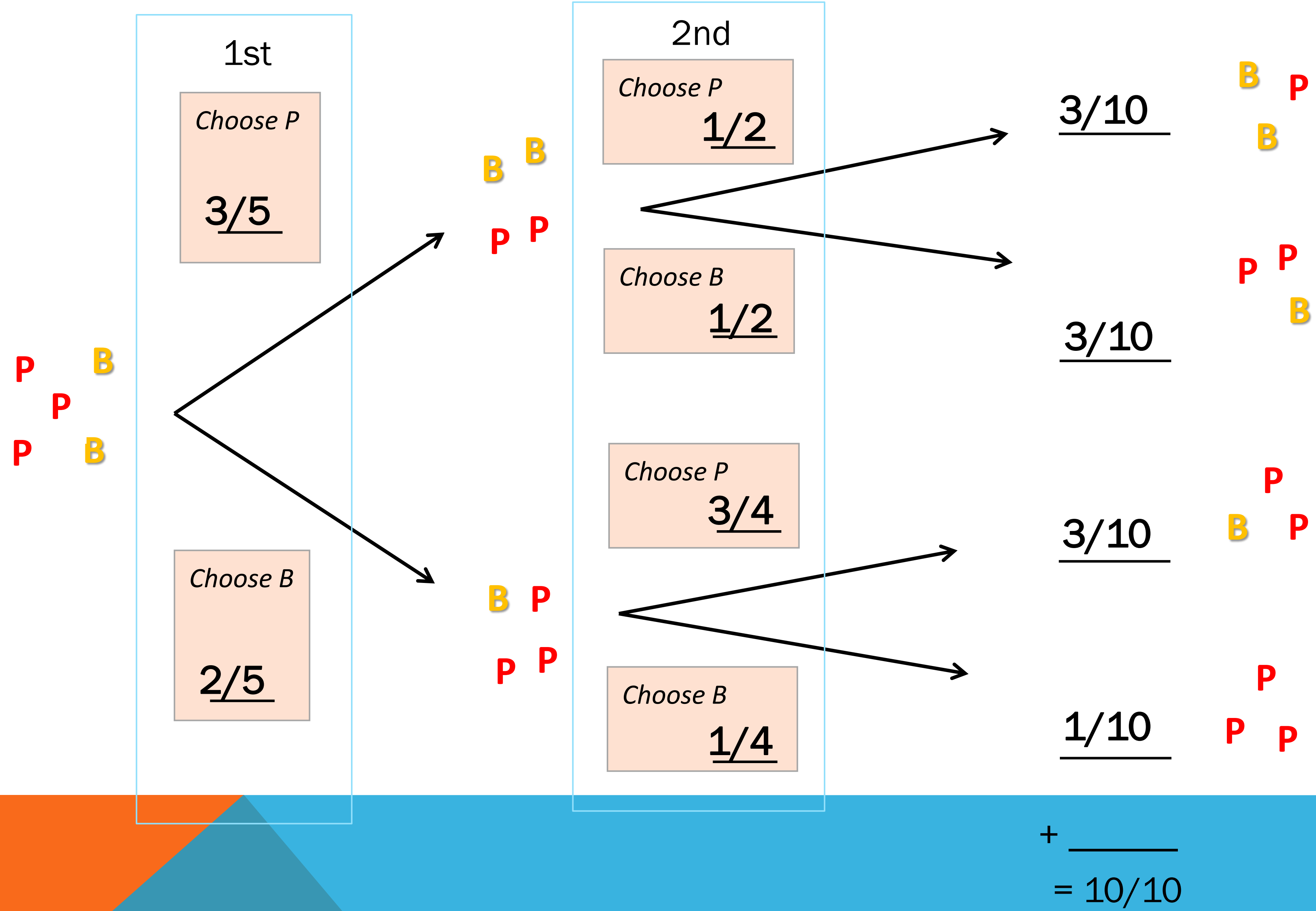
P(b and b) = 2/5 * 1/4

= 1/10

What is the probability that you chose a butterscotch and then a peppermint?

P(b and p) = 2/5 * 3/4

= 3/10



Tree Diagram method

What is the probability of choosing a jack or queen from a standard, 52 card deck?

Wait. This isn't an "AND" type of problem!!

THE "*OR*" RULE :

P(A *or* B) = P(A) + P(B)

P (j or q) = 4/52 + 4/52 = 8/52 → 2/13

You have a 2 in 13 chance of drawing a jack or a queen.

What is the probability of drawing a red or blue marker from the box that has 4 blue markers, 6 yellow, 2 black, and 8 red ones?

P (r or b) = 8/20 + 4/20

= 3/5

You have a 3 in 5 chance of drawing a red or blue marker.

Independent Event vs Dependent Event

What's the difference?

Independent: When the outcome of one event <u>doesn't</u> influence the outcome of the second event.

Dependent: When the outcome of one event <u>does</u> affect the outcome of the second event.

Which type did we experience when we chose one candy then ate it before the second draw?

Explain how you know.

Dependent. When you ate the first candy drawn, there were only 4 left instead of the original 5 for the second draw.

Conditional Probability:

The probability that Event B will occur if Event A has already happened.

$$P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}$$

At a middle school, 18% of all students play football and basketball and 32% of all students play football. What is the probability that a student plays basketball given that the student plays football?

What happened first?

football

$$P(B|A) = \frac{P(fb \,\&\, bb)}{P(fb)} \quad \frac{(.18)}{(.32)} = .5625$$

football

56.25% of the students play basketball given they play football.

70% of your friends like Chocolate, and 35% like Chocolate AND like Strawberry. What percent of those who like Chocolate also like Strawberry?

What happened first?

chocolate

$$P(B|A) = \frac{P(c \,\&\, s)}{P(c)} \quad \frac{(.35)}{(.70)} = .5$$

chocolate

50% of the students like chocolate given they like strawberry .

A jar contains black and white marbles.

Two marbles are chosen without replacement. The probability of selecting a black marble and then a white marble is 0.34, and the probability of selecting a black marble on the first draw is 0.47.

What is the probability of selecting a white marble on the second draw, given that the first marble drawn was black?

What happened first?

black

$$P(B|A) = \frac{P(b \,\&\, w)}{P(b)}$$

black

$$\frac{(.34)}{(.47)} = .7234$$

72.34% will select a white marble if a black marble is drawn first.

What happened first?

allowance

In the United States, 66% of all children get an allowance and 31% of all children get an allowance and do household chores. What is the probability that a child does household chores given that the child gets an allowance?

$$P(B|A) = \frac{P(a \,\&\, c)}{P(a)}$$

$$\frac{(.31)}{(.66)} = .4697$$

46.97% children do chores if they also get an allowance.

You try.

In Europe, 88% of all households have a television.
51% of all households have a television and Netflix.
What is the probability that a household has Netflix given that it has a television?

about **58%**

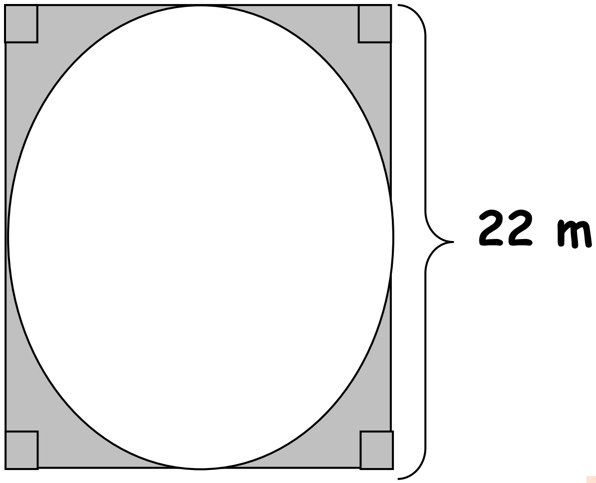
In New England, 81% of the houses have a garage and 65% of the houses have a garage and a back yard. What is the probability that a house has a backyard given that it has a garage?

about **80%**

At Kennedy Middle School, the probability that a student takes Technology and Spanish is 0.087. The probability that a student takes Technology is 0.68. What is the probability that a student takes Spanish given that the student is taking Technology?

about **13%**

DAY 4: GEOMETRIC PROBABILITY



Find the probability that a random point on the figure is in the shaded region.

Write your answer as a percent rounded to the nearest hundredth.

Plan: Area of Square – Area of Circle

Square Area = side^2

$= 22^2$

$= 484\ m^2$

Circle Area = πr^2

$= \pi\ 11^2$

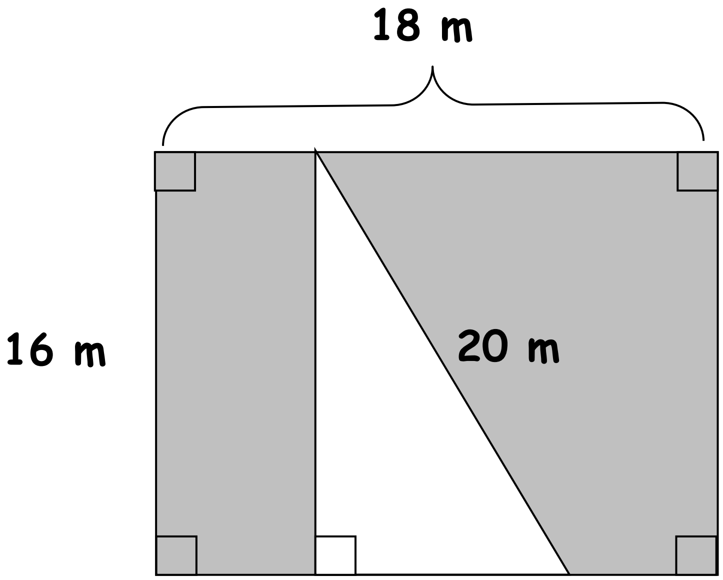
$= 380.13\ m^2$

This is only the shaded area !

484 – 380.13 = **103.87**

$$P = \frac{\text{shaded area}}{\text{total area}} = \frac{103.87}{484} = .2146 = 21.46\ \%$$

21.46% chance that a random point selected lies in the shaded region.



Find the probability that a random point on the figure is in the shaded region.

Write your answer as a percent rounded to the nearest hundredth.

Plan: Area of Rectangle – Area of Triangle

Rect. Area = 16*18

= 288 m^2

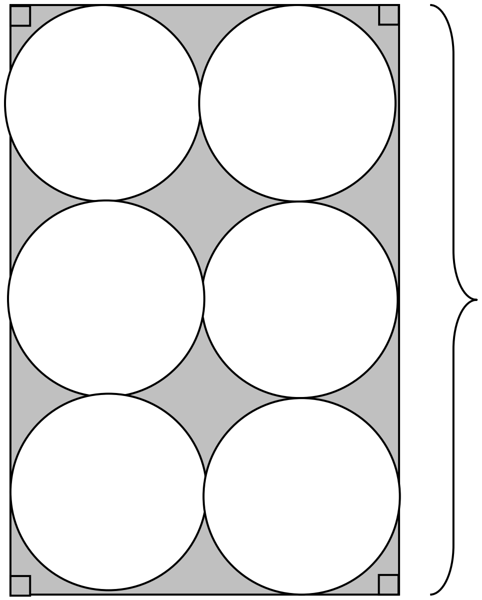
Tri. Area = (16*12) / 2

= 96 m^2

288 – 96 = **192** *shaded area*

$$P = \frac{\text{shaded area}}{\text{total area}} = \frac{192}{288} = .6667 = 66.67\ \%$$

66.67% chance that a random point selected lies in the shaded region.



Find the probability that a random point on the figure is in the shaded region.

Write your answer as a percent rounded to the nearest hundredth.

Plan: Area of Rectangle – Area of 6 circles

Rect. Area = 30* 20

= 600 in²

6 Circle Area = $\pi (5)^2$

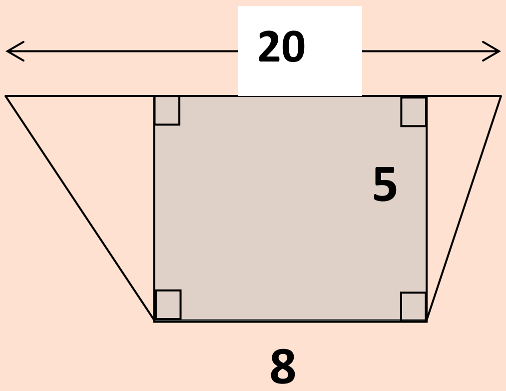
= 78.54 in² * 6

= 471.24 m²

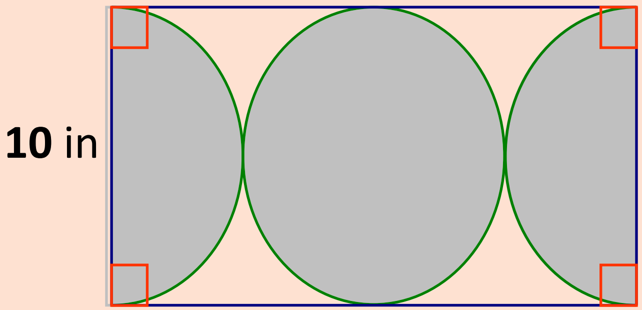
600 - 471.24 = **128.76** *shaded area*

$$P = \frac{\text{shaded area}}{\text{total area}} = \frac{128.76}{600} = .2146 = 21.46\ \%$$

21.46% chance that a random point selected lies in the shaded region.



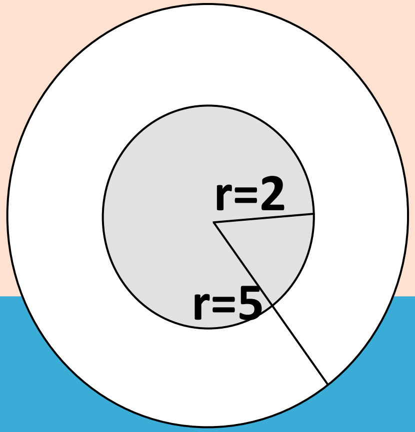
= 42.86 %



= 78.54 %

Try these!

Think ....



= 16 %